Tutorial 8
Advanced Graph Theory

September 18, 2013

1. There are n trips, t_1,...,t_n. Trip t_i has capacity n_i. Each person likes some trips and will travel on at most one trip he/she likes. Find the necessary and sufficient condition that fills all trips to capacity.
2. Give an example of three graphs of the same order,size and degree sequence such that no two of them are isomorphic. [8 marks]
3. Prove or disprove: If a graph H is obtained by applying any number of edge contractions to a simple Euler graph G, then H is also an Euler graph. Note that by definition of an edge contraction, H may have multiple edges between two vertices [6 marks]
4. Prove or disprove: If G is an n-vertex graph with maximum degree $\lceil \frac{n}{2} \rceil$ and minimum degree $\lfloor \frac{n}{2} \rfloor - 1$, then G is connected. [12 marks]

5. Find a maximum weighted matching of the following bipartite graph. (Rows and columns represent the vertices in the two partite sets respectively, and $X[i, j]$ represents the weight of the edge between node i $(0 \leq i \leq 4)$ in one partite set to node j $(0 \leq j \leq 4)$ in the second partite set). At each step, clearly show the equality subgraph, the matching/vertex cover found, and the updated cover. [15 marks]

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 7 & 2 \\ 1 & 3 & 4 & 4 & 5 \\ 3 & 6 & 2 & 8 & 7 \\ 4 & 1 & 3 & 5 & 4 \end{pmatrix}$$